COMPUTATION OF ORBIT OF THE COMES OF SIRIUS.

By A. W. Roberts.

[Read 25th November, 1891.]

I.—Introductory.

The orbit of any body moving in a curvilinear orbit can be determined either by a graphical or by an analytical method.

In a case of a body whose co-ordinates, either with reference to the earth, or the superior body round which it revolves, can be determined with very great precision, there can be no hesitation as to the method of procedure.

And yet even as an important auxiliary to the rapid determination of the orbit of a comet from early observations the graphical method merits our favour. When later positions are obtained these approximate positions can be corrected by the "variation of curtate distances."

With regard to binary stars the graphical method, for many reasons, is to be preferred.

The latitude of error precludes the more exact refinements of analytical research. We are therefore compelled to accept what *seems* the most probable orbit, and the computer strives to obtain an orbit that shall best agree with the observed measurements.

The following is an attempt to construct an orbit for Sirius that will not depart much from the observed path. The measures used are those collated by Mr. Gore, and published, along with an orbit, in the June number of the Monthly Notices of the Astronomical Society, 1889.

The orbit computed in this paper differs in one or two particulars from the one given by Mr. Gore. In both orbits the periodic time is about fifty-eight years. The Comes of Sirius according to this paper is at its minimum distance, *two seconds*, in 1892 ; according to Mr. Gore's orbit in 1892, the two stars are *three and a half seconds* apart, the peri-astron date being 1896.

The difference is mainly owing to the much greater eccentricity obtained in this paper.

It would be presumptuous to offer an opinion as to the merits of either orbit. When the pair opens out in 1896 an orbit will be obtained that will be worthy of more reliance than any yet computed.

In this paper most of the detailed working is left out, and only the more important steps given.

II.—Construction of Apparent Ellipse.

After all markedly erroneous measures have been set aside the observations of each year are combined. In the case of Sirius the following mean places were thus obtained :

Date.	Position Angle.	Distance.
	°	″
1862·22	84·72	10·25
1863·18	80·95	10·42
1864·22	79·60	10·92
1865·23	77·06	10·73
1866·23	74·68	10·60
1867·23	72·27	10 98
1868·25	70·63	11·15
1869·20	68·76	11·17
1871·22	64·15	10·92
1872·21	61·29	11·28
1873·22	60·84	10·57
1874·14	58·03	11·39
1875·21	56·68	11·10
1877·21	53·11	11·15
1877·98	52·24	10·77
1879·40	48·60	10·36
1880·14	48·95	9·99
1881·18	45·05	10·05
1882·13	42·85	9·58
1883·15	39·79	9·36
1884·18	36·59	8·82
1885·20	33·87	8·02
1886·14	29·41	7·35
1887·14	24·39	6·79
1888·97	13·85	5·27

These measures are then plotted down, and the interpolating curve drawn with as much care as possible. This curve ought to be a segment of an ellipse.

The centre is found by the property of the ellipse that if tangents be drawn at the extremities of any chord the line passing through the point of intersection of the tangents and the centre of the chord will also pass through the centre of the ellipse.

As all the lines *will not* pass through one point, the central point of all their intersections will probably be the centre.

The next step is to find *the position of the Axis Major.* In the case of Sirius, the form of the interpolating curve indicates that a portion of it lies on each side of this line. The line, therefore, from which opposite ordinates are equal, is the axis major.

The axis major being drawn, and the centre having already been found, *the focus* can be determined either from the equation,

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2),$$

or by trial with a pair of compasses.

III.—Transformation of Co-Ordinates.

The following measures are then carefully taken with a pair of compasses :

(1)	Semi-axis major...	7·″45
(2)	Semi-axis minor...	5·″27
(3)	Co-ordinate of centre of apparent ellipse along E. line (y)	4·″158
(4)	Co-ordinate along N. line (x)	0·″571
(5)	Inclination of Axis Major to Declination Circle	41·°30

Let the co-ordinates of the apparent ellipse be, referred to apparent centre, x' and y'.

Let the co-ordinates of the apparent ellipse, referred to Sirius, be x and y.

Let co-ordinates (3) and (4) be (α) and (β).

Let θ = inclination of axis major to E. and W. line ; then

$$x' = \cos\theta\,(x - \alpha) - \sin\theta\,(y - \beta)$$
$$y' = \sin\theta\,(x - \alpha) + \cos\theta\,(y - \beta)$$

But $x = \frac{b}{a}\sqrt{a^2 - y^2}$ (remembering that x is along N. axis), substituting and giving numerical values to the several constants we have :

$$(5{\cdot}27)^2 - ({\cdot}663x - {\cdot}749y + 2{\cdot}7358)^2 = ({\cdot}749x - {\cdot}663y - 3{\cdot}1844)^2\left(\frac{527}{745}\right)^2$$

Simplifying this equation we have :

$$-{\cdot}08152\,x + {\cdot}40828\,y - {\cdot}04736\,x^2 + {\cdot}03262\,xy - {\cdot}05133\,y^2 + 1 = 0$$

As a check upon these values we make $y = 0$.

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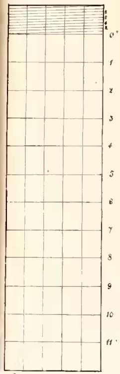
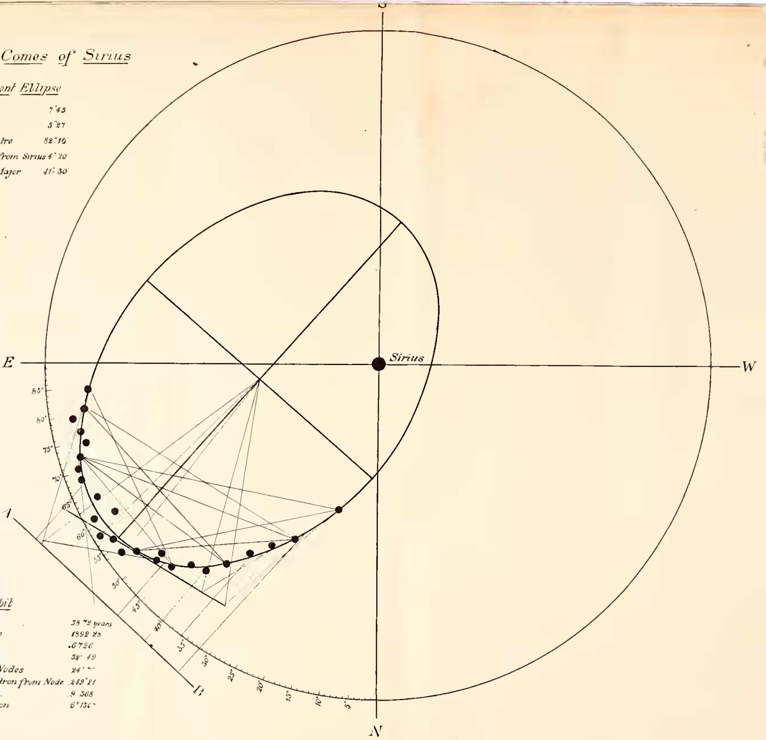
Apparent Orbit of Comes of Sirius

Elements of Apparent Ellipse

Semi Axis Major	7".45
Semi Axis Minor	5".27
Position Angle of Centre	82°10'
Distance of Centre from Sirius	4".20
Inclination of Axis Major	41°30'



Scale in Seconds



Elements of True Orbit

Periodic Time	58.72 years
Peri-Astron Passage	1892.25
Eccentricity	.6726
Inclination	52°49'
Position Angle of Nodes	24° [illegible]
Distance of Peri-Astron from Node	249°21'
Semi-Axis Major	9".368
Mean Annual Motion	6°.130

Then :

$$- \cdot 08152\, x - \cdot 04736\, x^2 + 1 = 0$$

And :

$$x = 3 \cdot 815 \text{ and } - 5 \cdot 535$$

Measurements by compass give,

$$x = 3 \cdot 81 \text{ and } - 5 \cdot 54$$

Making $x = 0$,

Then :

$$+ \cdot 40828\, y - \cdot 05133\, y^2 + 1 = 0.$$

And :

$$y = 1 \cdot 916 \text{ and } - 9 \cdot 920.$$

Measurements by compass give,

$$y = 1 \cdot 925 \text{ and } - 9 \cdot 920.$$

But the general equation to the ellipse is,

$$\alpha\, x + \beta\, y + \gamma\, x^2 + \delta\, x\, y + \varepsilon\, y^2 + 1 = 0$$

Therefore, substituting :

$$\alpha = - \cdot 08152,$$
$$\beta = + \cdot 40828,$$
$$\gamma = - \cdot 04736,$$
$$\delta = + \cdot 03262,$$
$$\varepsilon = - \cdot 05133.$$

IV.—Determination of all the Elements except Periodic Time and Peri-astron Date.

As already determined :

$\alpha = - \cdot 08152$	[8·91127] n
$\beta = + \cdot 40828$	[9·61097]
$\gamma = - \cdot 04736$	[8·67542] n
$\varepsilon = - \cdot 05133$	[8·71038] n
$\delta = + \cdot 03262$	[8·51356]

The following elegant formulae, given by Prof. Kowalsky, Kazan, determine the relations between these co-efficients and the elements of the true orbit :

$$\frac{\tan^2 i}{q^2} \sin 2\Omega = \delta - \tfrac{1}{2}\alpha\beta$$

$$\frac{\tan^2 i}{q^2} \cos 2\Omega = (\gamma - \varepsilon) - \tfrac{1}{4}(\alpha^2 - \beta^2)$$

$$\frac{2}{q^2} + \frac{\tan^2 i}{q^2} = - (\gamma + \varepsilon) + \tfrac{1}{4}(\alpha^2 + \beta^2)$$

$$\varepsilon \sin \lambda = - \frac{q}{2} (\beta \cos \Omega - \alpha \sin \Omega) \cos i$$

$$\varepsilon \cos \lambda = - \frac{q}{2} (\beta \sin \Omega + \alpha \cos \Omega)$$

$$a = \frac{q}{1 - \varepsilon^2}$$

The most important elements, Periodic time and Peri-astron passage, have to be determined by another method.

From these formulæ we deduce the following :

$$\delta - \tfrac{1}{2}\alpha\beta = \ldots + {\cdot}04926 = [8{\cdot}69249]$$

$$(\gamma - \varepsilon) - \tfrac{1}{4}(\alpha^2 - \beta^2) = \ldots + {\cdot}04398 = [8{\cdot}64325]$$

$$-(\gamma - \varepsilon) + \tfrac{1}{4}(\alpha^2 + \beta^2) = \ldots + {\cdot}14203 = [9{\cdot}15238]$$

Therefore :

$$\tan 2\Omega = \frac{{\cdot}04926}{{\cdot}04398} \qquad \begin{array}{r} 8{\cdot}69249 \\ 8{\cdot}64325 \\ \hline 0{\cdot}04924 \end{array}$$

$$2\Omega = 48^\circ - 14' \quad \ldots \quad \ldots \quad \Omega = 24^\circ - 7'$$

$$\frac{\tan^2 i}{q^2} = \frac{\delta - \frac{1}{2}\alpha\beta}{\sin 2\Omega} \qquad \begin{array}{r} 8{\cdot}69249 \\ 9{\cdot}87271 \\ \hline 8{\cdot}81978 \end{array}$$

$$= {\cdot}06604$$

$$\frac{2}{q^2} + \frac{\tan^2 i}{q^2} = -(\gamma + \varepsilon) + \tfrac{1}{4}(\alpha^2 + \beta^2)$$

$$\frac{2}{q^2} = {\cdot}14203 - {\cdot}06604 = {\cdot}07599$$

$$q^2 = \frac{2}{{\cdot}07599} \qquad \begin{array}{r} 0{\cdot}30103 \\ 8{\cdot}88076 \\ \hline 1{\cdot}42027 \end{array}$$

$$\tan^2 i \qquad \begin{array}{r} 1{\cdot}42027 \\ 8{\cdot}81978 \\ \hline 0{\cdot}24005 \end{array}$$

$$\tan i = 0{\cdot}12002 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad i = 52^\circ - 49'$$

$$(\beta \cos \Omega - \alpha \sin \Omega) \cos i = {\cdot}24536 = [9{\cdot}38980]$$

$$(\beta \sin \Omega + \alpha \cos \Omega) = {\cdot}09243 = [8{\cdot}96581]$$

$$\tan \lambda = \frac{(\beta \cos \Omega - \alpha \sin \Omega) \cos i}{\beta \sin \Omega + \alpha \cos \Omega}$$

$$= [9{\cdot}38980] - [8{\cdot}96581] = [0{\cdot}42399]$$

The negative value of sin λ shows us that λ is in the third quadrant ... ... $\lambda = 249^\circ - 21'$

$$\varepsilon = \frac{-q\,(\beta\cos\Omega - a\sin\Omega)\cos i}{2\sin\lambda}$$

= 0·71014n	0·30103
9·38980	9·97116n
0·09994n	0·27219n

0·27219n
9·82775 ... . ε = ·6726

$$a = \frac{q}{1-\varepsilon^2} \qquad (1 - \varepsilon^2) = [9{\cdot}73848]$$

0·71014
9·73848
0·97166 ... ... a = 9·368

V.—Determination of Peri-Astron Passage and Periodic Time.

The Peri-astron date and Periodic Time are computed from the following formulae :

$$(1)\quad \tan\psi = \cos i \tan(\theta + \lambda)$$

$$(2)\quad \tan\tfrac{1}{2}u = \sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\tan\tfrac{1}{2}\theta$$

$$(3)\quad u - \varepsilon\sin u = nt$$

The first formula gives the relation between the true orbit and its projection on the plane of the background of the heavens.

The second and third formulæ are the usual formulæ for elliptical motion in these expressions :

ψ = Angular distance from node.
i = Inclination of orbit.
θ = True anomaly.
λ = Longitude, or distance, of the Peri-astron from node.
u = Eccentric anomaly.
ε = Eccentricity of orbit.
n = Mean annual motion.
t = Time from Peri-astron date.

By the aid of these formulæ the position angles given in Table I. are reduced to mean angular distances from Peri-astron.

It is not necessary to reduce all the positions. Two sets a sfar